AN ACCURATE DESIGN OF RESONANCE FREQUENCIES OF DIELECTRIC RESONATORS.*

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ABSTRACT

The resonance frequencies of both cylindrical and rectangular dielectric resonators are obtained by a new method, where we assume that all the surfaces are imperfect magnetic walls. The theoretical values of the resonance frequencies have a good agreement with our experimental results with an error less than 1 percent.

INTRODUCTION

Since exact solutions of dielectric resonators having practical shapes other than a sphere cannot be rigourusly computed, approximate techniques must be adopted to solve the problem.

To show the principle of our method and in order to hold this paper's length within reasonnable bounds, we only detail here the case of dielectric disk resonator acting on the dipolar mode ; but we give the eigenvalue equations which are necessary to determine the resonance frequencies of the other modes of cylindrical and rectangular resonators.

Hence we have obtained curves which give rapidly the resonance frequencies of the resonators as a function of their parameters (dimensions, relative permittivity) if the resonator is isolated, and also as a function of the box's dimensions and substrate's permittivity and thickness if it is an element of a microstrip circuit.

The cylindrical resonator

. Isolated resonator : Let us consider a homogeneous, lossless, circular dielectric resonator of relative permittivity ε_2, of radius a and height H acting on the $TE_{0,n,p}$ modes.

We note :

I - the first order approximation : all the walls of the resonator are open circuit boundaries (O-C-B).

II_H - the second order approximation {1} : only the cylindrical surface satisfies the O.C.B. conditions, while at the flat surfaces the field extends outside. The resonance frequency for $TE_{0,n,p}$ modes can be obtained from :

$$H = \frac{2}{\beta} \tan^{-1} \frac{\alpha}{\beta} \qquad (1)$$

$$k_o = \omega \sqrt{\varepsilon_o \mu_o} \qquad k_c = \frac{x_{o,n}}{a}$$

$$\alpha^2 = k_c^2 - k_o^2 \qquad \beta^2 = k_o^2 \varepsilon_2 - k_c^2$$

For hybrid $EH_{m,n,p}$ modes we have a determinant (6 x 6). We don't give here its expression which is too complicated but you can found it in {2} . The characteristic equation is obtained from the requirement that this determinant vanishes.

II_A - we assume that the flat surfaces satisfy the O.C.B conditions, but the cylindrical surface does not. For $EH_{m,n,p}$ the characteristic equation is :

$$\left\{\frac{\varepsilon_2 . J'_n}{k_2 a . J_n} - \frac{K'_n}{k_1 a . K_n}\right\}\left\{\frac{J'_n}{k_2 a . J_n} + \frac{K'_n}{k_1 a . K_n}\right\} = \left\{\frac{n\beta'}{k_o} \frac{k_2^2 + k_1^2}{k_1^2\, k_2^2}\right\}^2 \qquad (2)$$

J_n, J'_n are function of $k_2 a$ and K_n, K'_n are function of $k_1 a$.

J_n Bessel's function of first kind : K_n : Bessel's function of second kind.

$$k_1^2 = \beta'^2 - k_o^2 \;,\; k_2^2 = k_o^2\, \varepsilon_2 - \beta'^2 \;,\; \beta' = \frac{p\Pi}{H}$$

Principle of our method

Our method consists of successive applications of these approximations, in order to determine an effective dielectric resonator : radius a_{eff}, height H_{eff}, permittivity ε_2. Then by applying the I and II_H approximation, we obtain two resonance frequencies, the combination of these gives the resonance frequency of the real resonator. To define the effective frequency we proceed as follows.

determination of a_{eff}	determination of H_{eff}
$(a,H) + II_H \rightarrow f_{1H}$	$(a,H) + II_A \rightarrow f_{2A}$
$(f_{1H},a) + I \rightarrow H_1$	$(f_{2A},H) + I \rightarrow a_1$
$(a,H_1) + II_A \rightarrow f_{1A}$	$(a_1,H) + II_H \rightarrow f_{2H}$
$(f_{1A},H_1) + I \rightarrow a_{eff}$	$(f_{2H},a_1) + I \rightarrow H_{eff}$

determination of f_{eff}

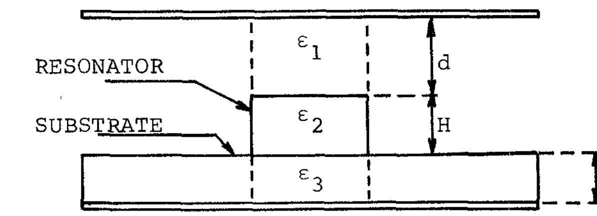
$$(a_{eff},H_{eff}) + I \rightarrow f_{1\,eff}, \quad (a_{eff},H_{eff}) + II_H \rightarrow f_{2\,eff}$$

$$f_{eff} = \frac{1}{2}\left(f_{1\,eff} + f_{2\,eff}\right)$$

With this method f_{eff} obtained is the best approximation of the resonance frequency of the real resonator. We present theoretical results on curve 1.

. Shielded cylindrical resonator

The resonator is in a microstrip structure



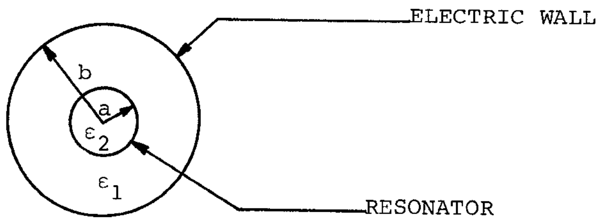
The resonance frequencies of $TE_{0,n,p}$ modes are given by : (4) (II_H approximation)

$$(\beta \tan\beta \frac{H}{2} \tanh \alpha_3 H - \alpha_3)(\alpha_1 \tan\beta \frac{H}{2} + \beta \tanh\beta\, d)$$
$$+ (\beta \tan \beta\frac{H}{2} \tanh \alpha_1 d - \alpha_1)\ (\alpha_3 \tan\beta \frac{H}{2} + \beta \tanh \alpha_3 h) = 0$$

$$\alpha_3^2 = k_c^2 - k_o^2\, \varepsilon_3 \qquad \alpha_1^2 = k_c^2 - k_o^2\, \varepsilon_1$$

we give the results of this approximation on curve 2.

The resonator is contained in a cylindrical waveguide



The eigenvalue is (5)

$$\left\{\frac{\varepsilon_2}{\varepsilon_1} k_1 J'_n(k_2a)\, W_1 - k_2 J_n(k_2a)\, X_1 \right\} \left\{k_1 J'_n(k_2a)\, Y_1 - k_2 J_n(k_2a).Z_1 \right\} - \frac{n^2 \beta'^2(k_2^2 - k_1^2)}{k_o^2.k_2^2.\varepsilon_1.\varepsilon_2.k_1^2} J_n^2(k_2a) W_1.Y_1 = 0$$

with :

$$W_1 = J_n(k_1b).Y_n(k_1a) - J_n(k_1a) \,.\, Y_n(k_1b)$$

$$Y_1 = J'_n(k_1b) \,.\, Y_n(k_1a) - J'_n(k_1a) \,.\, Y_n(k_1b)$$

$$X_1 = J_n(k_1b) \,.\, Y'_n(k_1a) - J_n(k_1a) \,.\, Y'_n(k_1b)$$

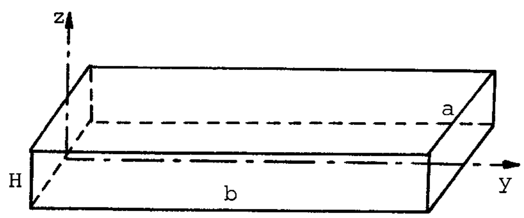
$$Z_1 = J'_n(k_1b) \,.\, Y'_n(k_1a) - J'_n(k_1a) \,.\, Y'_n(k_1b)$$

The curve 3 shows the chart we can obtain with such a geometry.

The rectangular resonator

. Isolated resonator

The method outlined above is also available to determinate the resonance frequencies of rectangular resonator.



. Modes $TE_{m,n,p}$

To determinate the II_H approximation we consider that the a and b surfaces satisfy O.C.B. conditions, while the field decay exponentially along the surfaces perpendicular to the propagation axis.

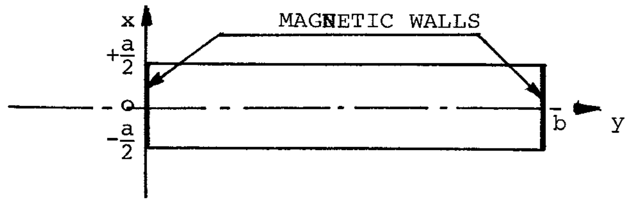
The eigenvalue equation (1) is still available but now :

$$k_c^2 = \Pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$$

The curve 4 gives the theoretical results obtained for a rectangular resonator acting on the TE_{11p} modes.

. Hybrid modes

More generally and for modes other than the TE or TM modes we can study the case for which only one surface of the section (a or b) satisfies the O.C.B. conditions.



Let the y=0, b and z = 0,H surfaces satisfy the O.C.B. conditions. The characteristic equation is :

$$\beta' k_y^2 \left\{\frac{1}{k_{c_1}^2} + \frac{1}{k_{c_2}^2}\right\} + k_o^2 \varepsilon_2 \left\{\frac{k_{x_2}^4}{k_{c_2}^4} + \frac{k_{x_1} k_{x_2}}{(k_{c_1} k_{c_2})^2} \tan k_{x_2} \frac{a}{2}\right\}$$

$$- k_o^2 \varepsilon_1 \left\{\frac{k_{x_1}^2}{k_{c_1}^4} + \frac{k_{x_1} k_{x_2}}{(k_{c_1} k_{c_2})^2} \frac{1}{\tan k_{x_1} \frac{a}{2}}\right\} = 0 \qquad (6)$$

$$k_y = \frac{m\Pi}{a} \quad , \quad \beta' = \frac{P\Pi}{H}$$

$$k_{x_2}^2 = k_o^2 \varepsilon_2 - (k_y^2 + \beta'^2) \qquad k_{x_1}^2 = k_y^2 + \beta'^2 - k_o^2 . \varepsilon_1$$

This relation (6) can be particularized to the case of L S E_x or L S M_x modes respectively longitudinal electric and magnetic fields

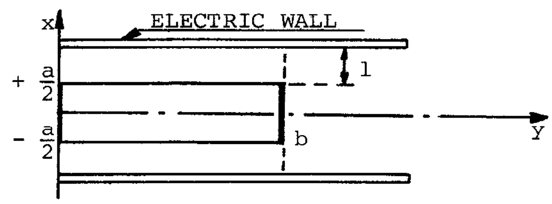
$$\text{L S } E_x \text{ modes} \; : \; a = \frac{2}{k_{x_2}} \tan^{-1} \frac{k_{x_1}}{k_{x_2}} \qquad (7)$$

$$\text{L S } M_x \text{ modes} \; : \; a = \frac{2}{k_{x_2}} \frac{1}{\tan^{-1} \frac{\varepsilon_2}{\varepsilon_1} \frac{k_{x_1}}{k_{x_2}}}$$

. shielded resonator

$TE_{m,n,p}$ mode resonator in a microstrip structure, the relation (4) is also available but it is necessary to remplace in this $\left(\frac{x_{On}}{a}\right)^2$ by $\Pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$ in the α_1 and α_3's values.

. hybrid modes of the resonator



The eigenvalue equation is (8)

$$\varepsilon_2 \frac{k_{x_2}^2}{k_{c_2}^4} + \frac{\varepsilon_1}{k_{c_1}^4} k_{x_1}^2 + \frac{k_{x_1} k_{x_2}}{k_{c_1}^2 k_{c_2}^2} \left\{ \frac{\varepsilon_1}{\tan k_{x_2}\frac{a}{2} \tan k_{x_1} l} \right\}$$

$$- \varepsilon_2 \tan k_{x_2} \frac{a}{2} \tan k_{x_1} l \} = 0$$

Experimental results

We give results for cylindrical resonator isolated and acting on the dipolar $TE_{0,1,p}$ modes in the table and for cylindrical resonators in a microstrip structure on the curve 2.

L mm	D mm	ε_2	f_{eff} MHz	experimental f f_{exp} MHz
7	10	38	4705	4660
8,4	15	38	3293	3315
5	15	66	2860	2850
7	10	66	3540	3506

A comparison shows that experimental and theoretical results agree within 1 %.

COHN has shown in his report {1} that to obtain the good resonance frequency by using the II_H approximation it is necessary to multiply the relative permittivity ε_2 by 0.875 . Our method confirms this value since we find a factor equal to 0.870.

CONCLUSION

This method allows to obtain with a great accuracy the resonance frequency of dielectric resonator, which is an important parameter for the development of microwave integrated filters.

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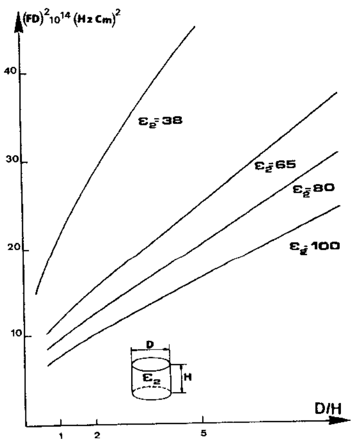

Fig.1 : Isolated cylindrical resonator

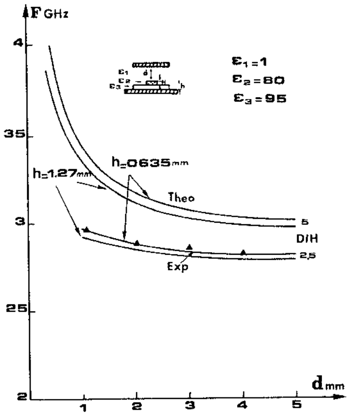

Fig.2 : Cylindrical resonator in a microstrip structure

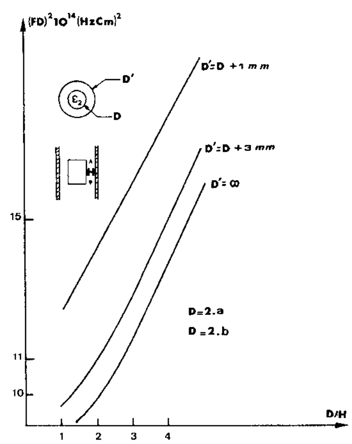

Fig.3 : Cylindrical resonator in an electric wall guide

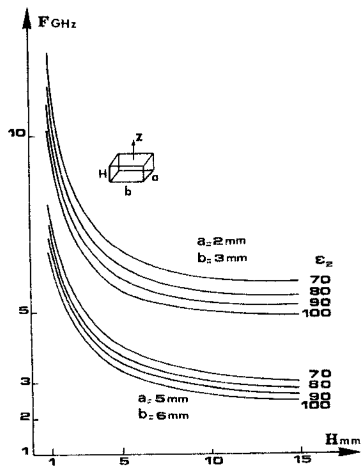

Fig.4 : Isolated rectangular resonator.